Predicate Refinement Heuristics in Program Verification with CEGAR

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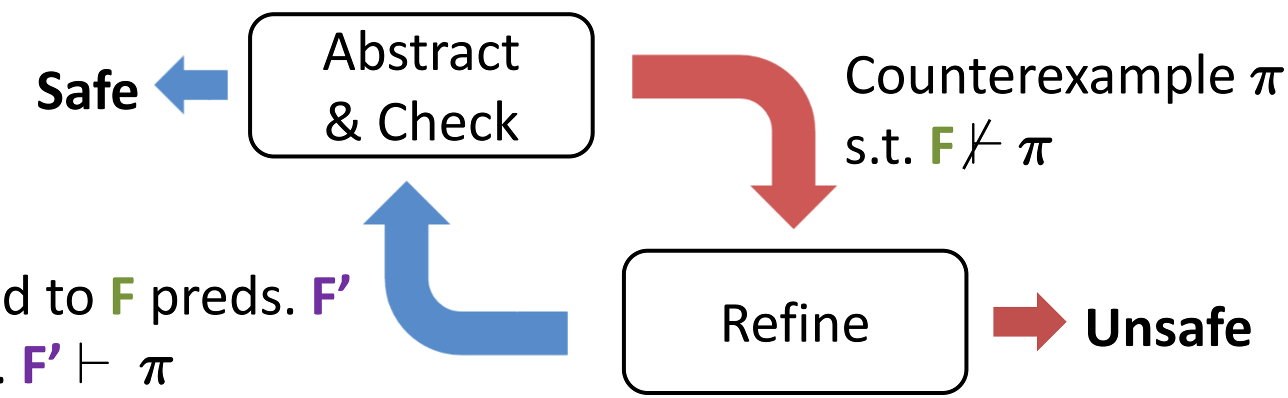
Predicate Abstraction with CEGAR

Iteratively generate **candidate predicates** F $\subseteq$ Preds(**T**)

- until F forms a proof of the given program
  - **T** : background FOL theory (e.g., QFLRA)



Much success for imperative programs (SLAM, BLAST, …)
for concurrent programs (Threader, SymmPA, …)
for functional programs (Depcegar, MoCHi, …)

Predicate Refinement

- Input:
  - Currently irrefutable counterexample π
    - i.e., F $\nvdash \pi$ where F is current candidate pred. set
- Output:
  - Set of predicates **F'** such that **F'** $\vdash \pi$

Issue:

- **There can be multiple (in general, ∞ many) F' s.t. F' $\vdash \pi$**
- **Choice of F' can significantly affect CEGAR performance**

Example

(How refinement choice affects CEGAR performance)

```
a = nondet(); b = nondet();
x = a; y = b; z = 0;
while (nondet()) {
  y++;z++;
}
while (z != 0) {
   y--;z--;
}
if (a=b && y!=x) { assert false; }
```

Counterexample π_1

```
a = nondet(); b = nondet();
x = a; y = b; z = 0;
if (nondet()) {
  y++;z++;
}
if (z != 0) {
  y--;z--;
}
if (a=b && y!=x) { assert false; }
```

Proof of the program:

- $\phi_{inv} \equiv a=b \Rightarrow y=x+z$

Proof of π_1: $\{ \phi_0, \phi_1, \phi_0 \vee \phi_1 \}$

- $\phi_0 \equiv x=a \wedge y=b \wedge z=0$
- $\phi_1 \equiv x=a \wedge y=b+1 \wedge z=1$

Sufficient for π_1 but not for the program

Example

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Proof of the program:

- $\phi_{inv} \equiv a=b \Rightarrow y=x+z$

π_1 : refuted by $\{ \phi_0, \phi_1, \phi_0 \vee \phi_1 \}$

π_2 : refuted by $\{ \vee F \mid F \subseteq \{\phi_0, \phi_1, \phi_2\} \}$

$\vdots$

π_i : refuted by $\{ \vee F \mid F \subseteq \{\phi_0, \ldots, \phi_i\} \}$

$\vdots$

$\pi_i \equiv$ Loops unfolded i times

$\phi_i \equiv x = a \wedge y = b + i \wedge z = i$

CEGAR DIVERGES!

Outline

- ✓ Introduction
2. Refinement scheme with convergence guarantee
3. Fast convergence via “small refinements”

Based on [Terauchi, Unno ESOP 2015]

REFINEMENT SCHEME WITH GUARANTEED CEGAR CONVERGENCE

Example

(How refinement choice affects CEGAR performance)

```
a = nondet(); b = nondet();
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π_2 : refuted by $\{ \vee F \mid F \subseteq \{\phi_0, \phi_1, \phi_2\} \}$

⋮

π_i : refuted by $\{ \vee F \mid F \subseteq \{\phi_0, \ldots, \phi_i\} \}$

⋮

$\pi_i \equiv$ Loops unfolded i times

$\phi_i \equiv x = a \wedge y = b + i \wedge z = i$

Proof of the program:

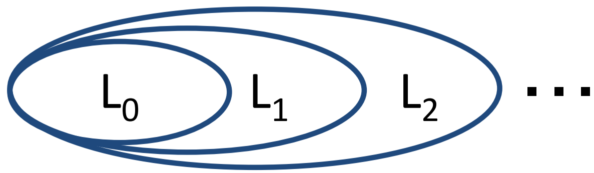
- $\phi_{inv} \equiv a=b \Rightarrow y=x+z$

Key Observation: ϕ_{inv} refutes every π_i

∴ Can force convergence by restricting predicates inferred by refinement

Stratified Refinement [1,2]

- Prepare growing strata of predicate sets:



  - Each $L_i \subseteq$ Preds(T) is finite
  - Preds(T) = $\bigcup_{i=1}^{\omega} L_i$
- In each refinement step:
  - Restrict inferred predicates to some L_i
  - raise L_i to next level when no proof of given c.e.x. is in L_i

GUARANTEED CEGAR CONVERGENCE

- **under promise that a proof exists in T**

[1] R. Jhala, K. McMillan. Practical and complete approach to predicate refinement. TACAS'06.
[2] K. McMillan. Quantified invariant generation using an interpolating saturation prover. TACAS'08.

Issue with Stratified Refinement

- Refinement step must decide if current L_i has a proof of given counterexample
  - i.e., decide if $\exists F \subseteq L_i \,.\, F \vdash \pi$
  - Such **exact finite-predicate-set-restricted proof search** is hard
    - cf. ESOP'15 paper for details

Our Goal

More Practical Refinement with Convergence Guarantee

– under the same promise that a proof exists in Preds(T)

New Proposal: Relaxed Stratification

- Prepare strata of predicate set pairs

$$B_0 \cup E_0,\ B_1 \cup E_1,\ \ldots\ B_i \cup E_i\ \ldots$$

Base & Extension

  - Each $B_i \cup E_i \subseteq Preds(T)$ is finite
  - $B_i \subseteq B_{i+1}$ for each B_i
  - $Preds(T) = \bigcup_{i=1}^{\omega} B_i$

(Exact) stratification is special case where $E_i = \emptyset$

- In each refinement step:
  - Restrict inferred predicates to some $B_i \cup E_i$
  - Fail to infer preds. and raise level only if no proof is in B_i

Need not to exactly decide existence of proof in B_i or in $B_i \cup E_i$

Correctness

Theorem: With relaxed stratification, CEGAR converges under the promise that program can be proved by Preds(T)

Proof sketch:

– Follows from **Key Observation**: proof of program is proof of its counterexamples. Therefore:
  - Stratum only goes up to $B_i \cup E_i$ where $B_i \supseteq$ proof of prog.
  - Stays in same stratum only for finite number of CEGAR iterations

Outline

✓ Introduction

2. Refinement scheme with convergence guarantee
   ✓ Relaxed Stratification Scheme
   b. Concrete instances of relaxed stratification
      • “Restricted” recursion-free Horn-clause solvers
   c. Experiments
3. CEGAR iteration bound via “small” refinement

Horn Clause Constraints

$$Predicate\ variables \quad P, Q, P_1, P_2, \ldots$$

$$\begin{array}{lrcl} Body\ clause & B & ::= & \theta \mid P(\vec{x}) \mid B \wedge B \\ Horn\ clause\ constraint & hc & ::= & B \Rightarrow P(\vec{x}) \mid B \Rightarrow \bot \\ Horn\ clause\ constraint\ set & H & ::= & \emptyset \mid H \cup \{hc\} \end{array}$$

- H is *satisfiable* if $\exists \sigma : pvars(H) \mapsto \mathtt{Preds}(\mathcal{T}). \forall hc \in H. \models \sigma(hc)$.
- H is *recursion-free* if $\{(P, Q) \mid \ldots P(\ldots) \cdots \Rightarrow Q(\ldots) \in H\}$ is acyclic.
- H is *tree-like* if each $P \in pvars(H)$ occurs at most once in left of $\Rightarrow$ and at most once in right of $\Rightarrow$.

Reducing Predicate Refinement to Horn-clause Solving

Prop: For any π, exists recursion-free H_π such that

$$\sigma \text{ is solution of } H_\pi \iff \{\sigma(P) \mid P \in dom(\sigma)\} \vdash \pi$$

Standard Refinement Algorithm:

1. Build H_π
2. Find solution σ of H_π
3. Return $\{ \sigma(P) \mid P \in dom(\sigma) \}$ as inferred predicates

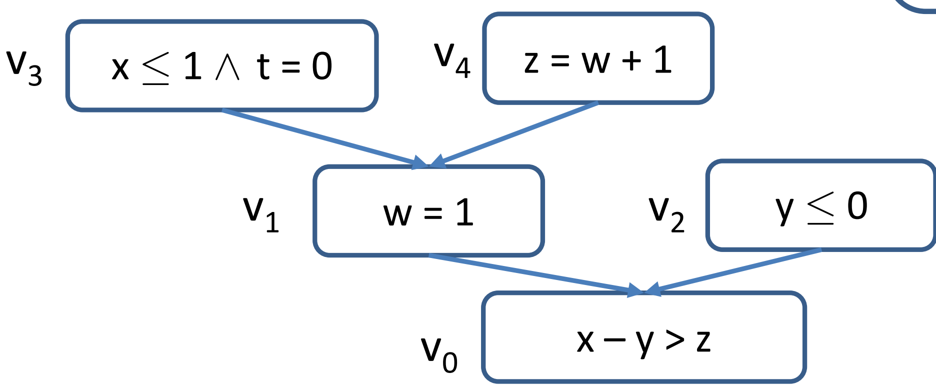
Plan: Modify Step 2 so that

- Finds solution from $B_i \cup E_i$ (**not from entire Preds(T)**)
- Fails to solve only if no solution is in B_i

Technical Detour: Tree Interpolation

- Labeled tree (V,E,Θ) - V: nodes E: edges $\Theta: V \to \text{Formula}(T)$
- $I: V \to \text{Formula}(T)$ is **tree interpolant (ITP)** of $(V,E,\Theta) \Leftrightarrow$
  - For root $v \in V$, $I(v) = \text{false}$
  - $\forall v \in V.\ \Theta(v) \wedge (\bigwedge_{(v,v') \in E} I(v')) \Rightarrow I(v)$
  - $\forall v \in V.\ \text{vars}(I(v)) \subseteq \text{sharedvars}(v)$

vars. occurring both in & out of subtree rooted at v



$I(v_4) = z \geq w + 1$

$I(v_3) = x \leq 1$

$I(v_2) = y \leq 0$

$I(v_1) = x \leq z$

$I(v_0) = \text{false}$

Reducing Horn clauses to Tree Interpolation

Theorem[Rümmer+'13]: Given tree-like clause set H, can build tree-interpolation instance (V_H,E_H,Θ_H) s.t.

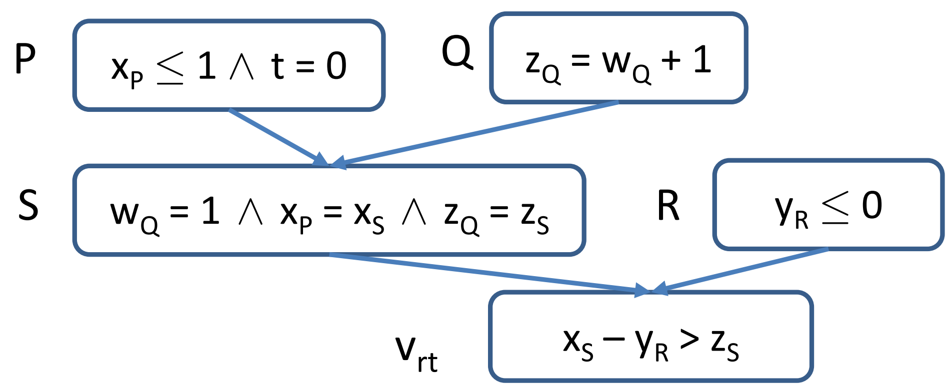
$$\{P \mapsto \lambda \overrightarrow{x_P}. I(P)\} \models H \Leftrightarrow I \text{ is tree-interpolant of } (V_H, E_H, \Theta_H)$$

$$\text{where } V_H = pvars(H) \cup \{v_{rt}\},\ v_{rt} \notin pvars(H), \text{ and } \overrightarrow{x_P}\text{'s are fresh variables.}$$

Can be extended to general recursion-free case (cf. [Rümmer+'13])

– Example:

$$\boxed{\begin{array}{l} x \leq 1 \land t = 1 \Rightarrow P(x) \\ z = x + 1 \Rightarrow Q(w, z) \\ y \leq 0 \Rightarrow R(y) \\ w = 1 \land P(x) \land Q(w, z) \Rightarrow S(x, z) \\ x - y > z \land S(x, y) \Rightarrow \bot \end{array}}$$



Review

- We want to:
  - Infer solution σ of H_π s.t. ran(σ) $\subseteq$ $B_i \cup E_i$
  - Fail to infer only when no solution of range B_i exists

- Horn-clause to tree interpolation reduction says:

$$\{P \mapsto \lambda \overrightarrow{x_P}.I(P)\} \text{ is solution of } H \Leftrightarrow I \text{ is tree interpolant of } (V_H, E_H, \Theta_H)$$

- So, it suffices to do “restricted” tree interpolation:
  - Infer tree interpolant I of (V_H, E_H, Θ_H) s.t. ran(I) $\subseteq$ $B_i \cup E_i$
  - Fail to infer only when no interpolant of range B_i exists

Restricted Tree Interpolation

Standard tree interpolation algorithm:

- Input: (V,E,Θ)
- Use SMT solver to check $\wedge_{v \in V}\Theta(v)$ is UNSAT
  - obtain resolution proof deriving "false"
- Compute **partial ITPs** at each node of resolution proof
  - ITP = partial ITP at root node

Modification: theory-level-restricted SMT solving

- Restrict leaf (i.e., theory) level reasoning to only produce partial ITPs in B_i
  - Prototype implementation using template-based technique

Only uses expensive finite preds. res. search at leaf levels

Tree Interpolant Generation

- p $\in$ Atoms(T)
- C ::= p | $\neg$p | C $\vee$ C

$$\frac{C \wedge \ldots = \Theta(v')}{(V, E, \Theta) \vdash C : I} \qquad I = \lambda v. \begin{cases} \texttt{false} & \text{if } (v', v) \in E^* \\ \texttt{true} & \text{otherwise} \end{cases}$$

$$\frac{(V, E, \Theta) \vdash p \vee C_1 : I_1 \quad (V, E, \Theta) \vdash \neg p \vee C_2 : I_2}{(V, E, \Theta) \vdash C_1 \vee C_2 : I} \qquad I = \lambda v. \begin{cases} I_1(v) \wedge I_2(v) & \text{if } p \in outs(v) \\ I_1(v) \vee I_2(v) & \text{otherwise} \end{cases}$$

$$\frac{\models_T C}{(V, E, \Theta) \vdash C} \Rightarrow \frac{I \text{ is tree itp. of } (V, E, \Theta_C) \text{ where } ran(I) \subseteq B_i}{(V, E, \Theta) \vdash C : I}$$

Θ_C = Θ with labels restricted to Atoms(C)

Theorem: Inferred ITP is in $B_i^{\wedge\vee}$, and some ITP is inferred if one exists in B_i

So, $E_i = B_i^{\wedge\vee}$

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- ✓ Introduction
2. Refinement scheme with convergence guarantee
   - ✓ Relaxed Stratification Scheme
   - b. Concrete instances of relaxed stratification
     - "Restricted" recursion-free Horn-clause solvers
       - ✓ Theory-level-restricted tree interpolation
       - ii. Another concrete instance
   - c. Experiments
3. CEGAR iteration bound via "small" refinement

Another Concrete Instance of Relaxed Stratification Scheme

- “Restricted” Horn clause constraint solver (again)

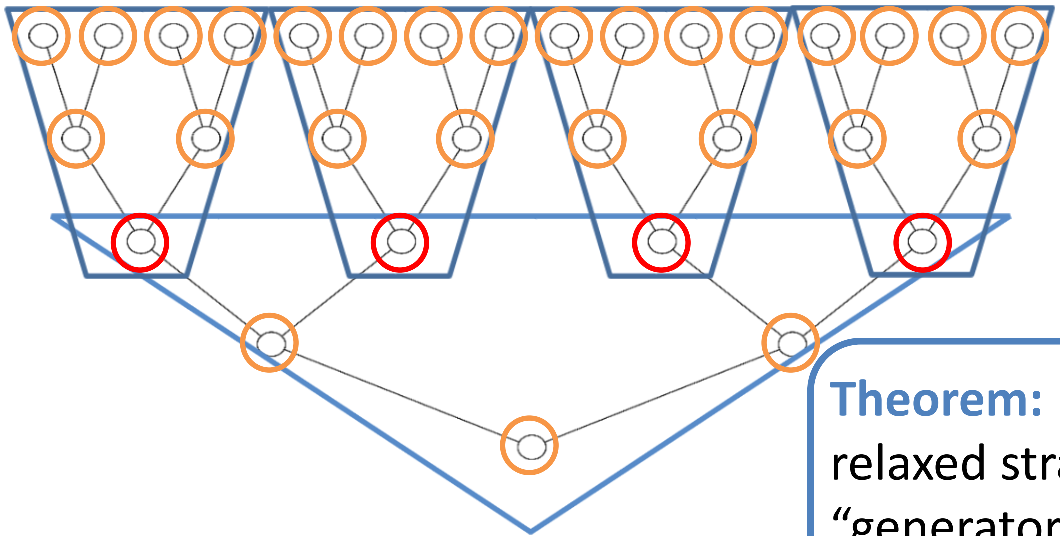
  Two-phase approach

  1. Partition clauses into bounded-size trees
  2. Infer restricted solutions to predicate variables at partition boundaries
  3. Infer unrestricted solutions to the rest

  Only use expensive finite predicate-restricted search for few predicate variables

  Becomes relaxed stratification under some mild assumptions on generated counterexample patterns

Algorithm Explained Pictorially



Theorem: This satisfies the requirements of relaxed stratification, assuming there is a "generator" clause set H_{gen} s.t. any c.ex. is an unfolding of H_{gen}

1. **Partition into bounded-size sub-trees**
2. **Infer restricted solutions to boundary predicate variables**
   - E.g., via template-based method
     - Could also use another relaxed stratification refinement alg. itself
3. **Infer unrestricted solutions to the rest**
   - via standard approaches [Unno+'09,Terauchi'10,Rümmer+'13, etc.]

Refinement Algorithm Schemas

- These are actually **algorithm schemas**
  - take other refinement algorithms as modules
  - generate refinement algorithms satisfying requirements of relaxed stratification

See ESOP’15 paper for details

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     - ✓ “Restricted” Horn clause solvers algorithms
       - ✓ Theory-level-restricted tree interpolation
       - ✓ 2-phase inference approach via bounded partitioning
   - c. Experiments
3. Fast convergence via “small refinements”

Prototype Implementation

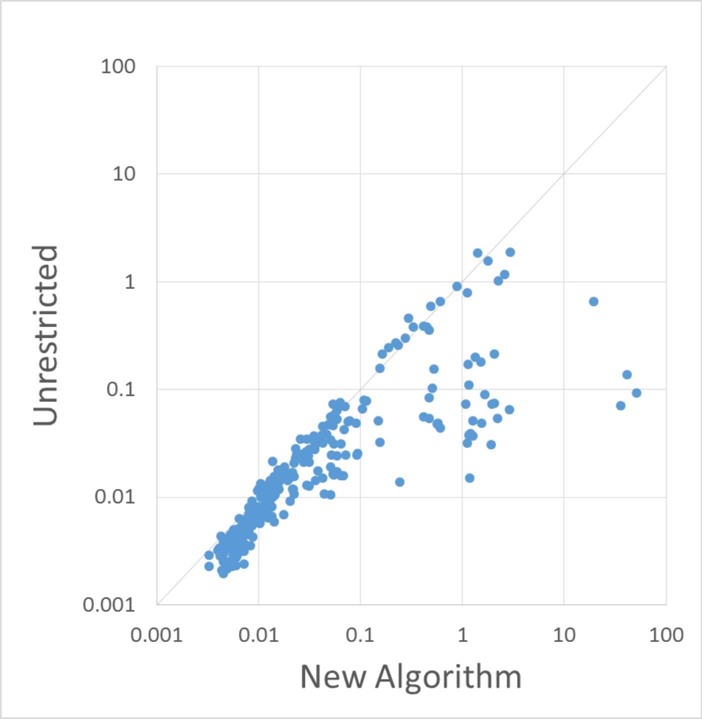
New refinement algorithm

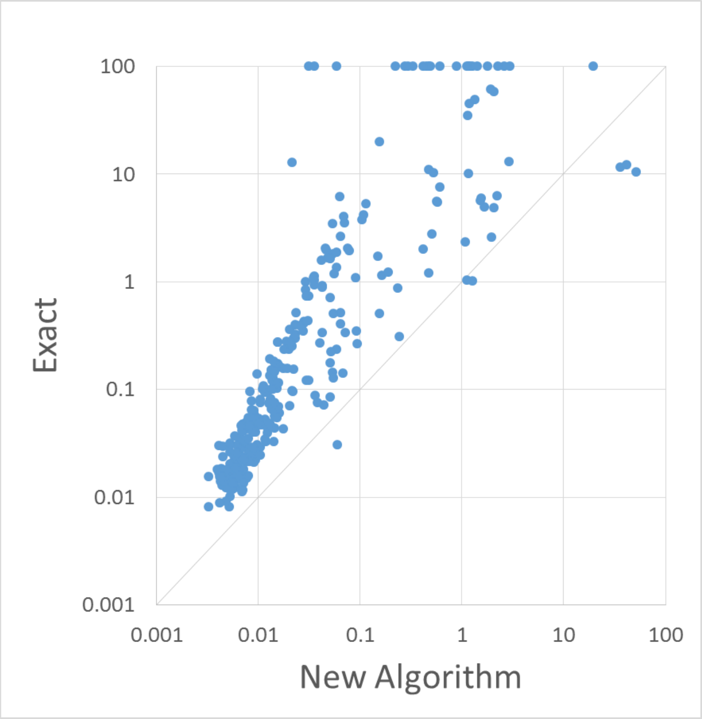
- Algorithm 1 used as module of algorithm 2
- Z3 [1] for template-based constraint solving
- MathSAT5 [2] for unrestricted refinement

- Used as refinement engine of MoCHi [3]
  - Software model checker for higher-order functional programs based on CEGAR

[1] http://z3.codeplex.com/
[2] http://mathsat.fbk.eu/
[3] http://www.kb.is.s.u-tokyo.ac.jp/~ryosuke/mochi/

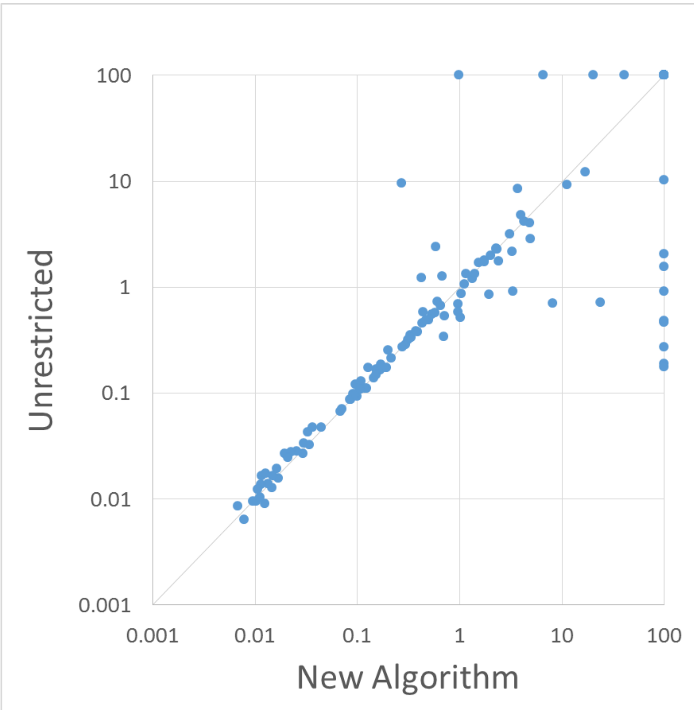
Experiment Results: Individual Refinement Runs

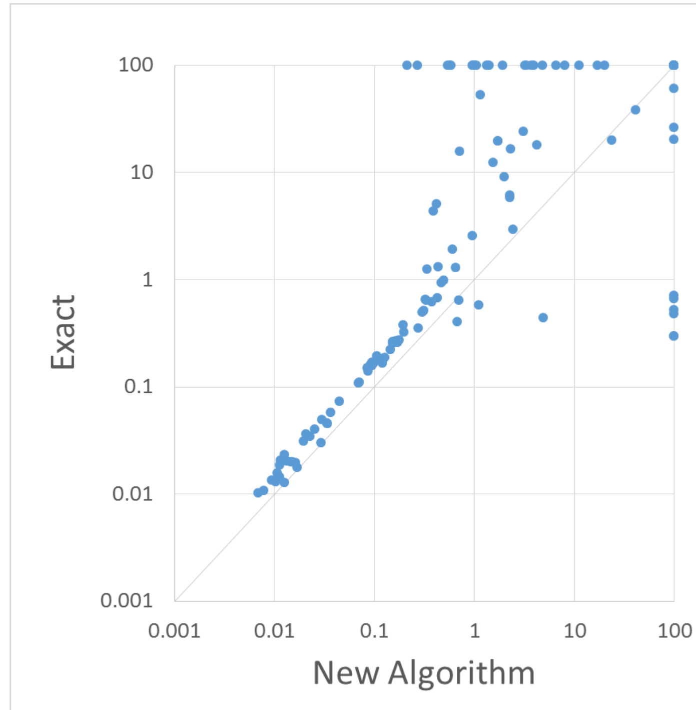




- 318 counterexamples generated from 139 benchmark programs
- Three refinement algorithms:
  - New algorithm
  - Unrestricted refinement
  - Exact stratification

Experiment Results: Overall Verification Performance





- 139 benchmark programs
- MoCHi with each refinement algorithm:
  - New algorithm
  - Unrestricted refinement
  - Exact stratification

Outline

- ✓ Introduction
- ✓ Refinement scheme with convergence guarantee
  - ✓ Relaxed Stratification Scheme
  - ✓ Concrete instances of relaxed stratification
    - ✓ “Restricted” Horn clause solvers algorithms
  - ✓ Experiments

3. Fast convergence via “small refinements”

Based on [Terauchi SAS 2015]

FAST CONVERGENCE VIA “SMALL REFINEMENTS”

Talk so far

Predicate refinement in CEGAR

- Return predicates that refutes given counterexample
- Clever choice of predicates can make CEGAR converge

Q: Can we say anything about convergence speed?

Short Answer: YES

Our Result

Small Refinement Heuristic (SRH)

- **Refinement phase returns "small" proof of counterexample's safety**

  [Hoder+'12][Scholl+'14][Albarghouthi,McMillan'14][Unno,Terauchi'15]

- **We will show:**
  - CEGAR with SRH converges in number of CEGAR iterations bounded in the size of the proof for the input program

Example (from before)

```
a = nondet(); b = nondet();
x = a; y = b; z = 0;
while (nondet()) {
  y++;z++;
}
while (z != 0) {
   y--;z--;
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if (a=b && y!=x) { assert false; }
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π_2 : refuted by $\{ \vee F \mid F \subseteq \{\phi_0, \phi_1, \phi_2\} \}$

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$\pi_i \equiv$ Loops unfolded i times

$\phi_i \equiv x = a \wedge y = b + i \wedge z = i$

Proof of the program:

$\phi_{inv} \equiv a=b \Rightarrow y=x+z$

Key Observation: ϕ_{inv} refutes every π_i

$\therefore$ **Inferring small refinements should hasten convergence**

Refinement will infer ϕ_{inv} or some other small proof of program before inferring large ϕ_k's

How to define "small"?

Proof Size Metric and SRH

Def: size : $\mathcal{P}_{\text{fin}}$(Preds(T)) $\rightarrow$ Nat is **generic proof size metric** if $\exists$c>0.$\forall$n$\geq$0. $|\{ F \mid \text{size}(F) \leq n \}| \leq c^n$

Def: **minprfsize**(γ) = $\min_{F \in \{ F \mid F \vdash \gamma \}}$ size(F)

γ : cex or program

Def: CEGAR with **Small Refinement Heuristic (SRH)** is CEGAR with Refine() satisfying:

$\exists$poly f. $\forall\pi$. if Refine(π) = F then size(F) $\leq$ f(minprfsize(π))

Convergence Bound

Theorem: Suppose proof size metric is generic. Then CEGAR with SRH converges in number of iterations bounded exponentially in minprfsize(P).

– **Proof: By KEY OBSERVATION and simple counting argument**

Corollary: CEGAR with SRH converges in exponential number of iterations under the promise that program has polynomial size proof

Can a tighter bound be obtained with a more concrete setting?

Bound for CFG-represented Programs

- Assumptions
  - Program represented by reducible Control Flow Graph
  - Counterexamples are loop-unfoldings of the CFG
    - every loop unfolded the same number of times
  - Proof is Floyd-style node-wise inductive invariant
  - Predicate abstraction is Cartesian predicate abstraction
  - size(F) = $\sum_{\phi \in F}$ syntactic_size(ϕ)
- **Theorem: CEGAR with SRH converges in number of iterations bounded polynomially in minprfsize(P) for CFG programs.**

See SAS'15 paper for details

Outline

- ✓ Introduction
- ✓ Refinement scheme with convergence guarantee
  - ✓ "Relaxed" Stratification
3. Fast convergence via "small refinements"
   - ✓ Generic Setting
     - ✓ exp(minprfsize(P)) CEGAR iteration bound
   - ✓ CFG-represented Programs
     - ✓ poly(minprfsize(P)) CEGAR iteration bound (under various assumptions)

Some Thoughts and Future Work

Results on CEGAR iteration bound can be taken as negative results?

- Inferring small refinements must be hard because otherwise verification would be easy?
  - Substantiates the experience with stratified refinement

Need further investigation

- Hardness of verification w.r.t. “predicate refinement oracles” and under “small proof promise” seems to be underexplored